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THE STABILITY OF AGGLOMERATIONS OF MICROPARTICLES IN THE GRAVITATIONAL-REPULSIVE FIELD OF BINARY STELLAR SYSTEMS†

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A physically clearer analysis of the stability of the positions of relative equilibrium (triangular and collinear libration points) of micrometeorite particles of gas-dust clouds in the field of two gravitating and radiating starts is presented, based on the circular restricted three-body problem. The analysis is new compared with that of [1–3]. By introducing a new parameter, characterising the gravitational-repulsive field of the system, and changing to configuration space, a complete picture of the regions of stability of triangular and collinear libration points is established and also their evolution over the whole range of variation of the fundamental parameters of the system. The stability regions constructed enable one to judge possible stable configurations of agglomerations of micrometeorite and gas-dust particles in binary stellar systems. © 2003 Elsevier Ltd. All rights reserved.

A new (and much clearer) analysis of the stability of families of positions of relative equilibrium (libration points) of micrometeorite particles (or the particles of a gas-dust cloud) in the gravitational-repulsive field of two bodies (stars), which rotate about their baricentre in Kepler orbits, was presented in [4, 5]. An analysis of the stability of families of collinear libration points was made in [5], but only for the special case of an identical repulsive effect on a particle from the side of both radiating bodies. The purpose of the present paper is both to remove this limitation and also to establish a relation between the stable families of collinear libration points and the families of stable triangular libration points, and to clarify the whole picture of their evolution over the whole possible range of variation of system parameters. It is possible to obtain a fairly simple and physically clear picture of the stability regions and their evolution by using the method of parameter elimination, the effectiveness of which has been demonstrated previously [6] when solving problems similar to the one considered here. The advantages of this method of constructing stability regions lies both in the possibility of obtaining an analytical solution in cases when the traditional approach does not enable this to be done, and in constructing stability regions directly in configuration space, which always gives a clearer picture of the stable states of the system.

As was shown in [4, 5], the coordinates of the libration points and the conditions for their stability are defined both by the parameters of the gravitational-repulsive field and by the windage of the particles situated in this field, representing the ratio of the characteristic area of the particle to its mass. We must take as the parameter characterizing the gravitational-repulsive field of the chosen stellar pair, the quantity defined as [4, 5]

$$k = (M_2/C_2)/(M_1/C_1)$$

where M_1 and M_2 are the masses of the main bodies, and C_1 and C_2 are their radiation powers respectively. Obviously, k can take any non-negative values.

The force function of the problem considered (also called the restricted photogravitational three-body problem) has the form [4, 5]

$$\begin{aligned} W &= (x^2+y^2)/2 + Q_1(1-\mu)/R_1^2 + Q_2\mu/R_2^2 \\ R_i &= (x^2-a_i)^2 + y^2 + z^2, \quad i = 1, 2 \end{aligned} \tag{1}$$

where x, y, z are the dimensionless rectangular coordinates of a passively gravitating particle P in the $Oxyz$ system, rotating uniformly about the z axis with angular velocity equal to unity, μ and $1-\mu$ are the dimensionless masses of the main bodies S_2 and S_1, referred to their common mass, $a_1 = -\mu$ and

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$a_2 = 1 - \mu$ are their dimensionless coordinates, and Q_1 and Q_2 are mass reduction factors for the bodies S_1 and S_2, characterising the effect of the repulsive field of the light pressure and representing the ratio of the difference of the gravitational and repulsive forces to the gravitational force. As was shown in [4, 5], for any field value of k (i.e. for a fixed pair of the main bodies S_1 and S_2) the mass reduction factors cannot be arbitrary, but must be related by the following linear rotation

$$(1 - Q_2)/(1 - Q_1) = k \tag{2}$$

which has been ignored in the majority of previous papers [7]. This has not enabled correct conclusions to be drawn regarding the disposition and stability of the liberation points as a function of the windage of the particles for any fixed pairs of main bodies. In this paper we investigated the stability of collinear libration points for all permissible values of the system parameters k and μ ($0 \le k < \infty$, $0 < \mu \ge 1/2$).

As has been shown in a number of previous papers [1–3, 5], external libration points (lying outside the segment S_1S_2) are unstable for all permissible values of k and μ, and hence we will only consider internal libration points, lying on the segment S_1S_2. Then, as has been shown previously [1, 2], the stability of these points is only possible as a result of gyroscopic stabilization, since the potential energy of the system cannot have a minimum either for triangular libration points or for collinear libration points.

The condition for such gyroscopic stabilization for collinear libration points has the form [1]

$$8/9 \le A \le 1, \quad A = (1-\mu)Q_1/R_1^3 + \mu Q_2/R_2^2 \tag{3}$$

and is obtained from the requirement that there should be no real parts of the roots of the characteristic equation of the linearized system of equations of the perturbed motion.

Using the equation of equilibrium (obtained from the condition $\delta W = 0$ when $y = 0$)

$$x - \frac{1-\mu}{R_1^3}(x - a_1)Q_1 - \frac{\mu}{R_2^3}(x - a_2)Q_2 = 0$$

and relation (2) between Q_1 and Q_2 given above we will have

$$A = \left(\frac{1-\mu}{R_1^2} - k\frac{\mu}{R_2^2}\right)^{-1}\left[\left(\frac{1-\mu}{R_1^3} + k\frac{\mu}{R_2^3}\right)x + \frac{\mu(1-\mu)}{R_1^3R_2^3}(1-k)\right]$$

$$x = R_1 - \mu = 1 - \mu - R_2$$

We will introduce the following notation

$$f_1(R_1, R_2) = \frac{1-R_2^3}{1-R_1^3}, \quad f_2(\mu, R_1, R_2) = \frac{1-\mu}{\mu}\left(\frac{R_2}{R_1}\right)^2, \quad f_3(\mu, R_1, R_2) = \frac{\tilde{f}_3(\mu, R_1, R_2)}{\tilde{f}_3(1-\mu, R_1, R_2)}$$

$$\tilde{f}_3(\mu, R_1, R_2) = (1-\mu)[9\mu(1-R_2^3) + R_1R_2^3]$$

Taking into account the fact that, for internal libration points, $R_1 + R_2 = 1$, we obtain the following two pairs of inequalities from the condition $A \le 1$

$$k > f_2(\mu, R_1, R_2), \quad k \le f_1(R_1, R_2) \tag{5}$$

or

$$k < f_2(\mu, R_1, R_2), \quad k \ge f_1(R_1, R_2) \tag{6}$$

(the case $k = f_2(\mu, R_1, R_2)$ will be considered below).

Similarly from the condition $A \le 8/9$ we derive the following two pairs of inequalities

$$k < f_2(\mu, R_1, R_2), \quad k \le f_3(R_1, R_2) \tag{7}$$

$$k > f_2(\mu, R_1, R_2), \quad k \ge f_3(\mu, R_1, R_2) \tag{8}$$

We will analyse the inequalities obtained for different of $k \ne 1$. Curves of $k = f_1(R_1, R_2)$, $k = f_i(\mu, R_1, R_2)$ ($i = 2, 3$) in the k, R_1 plane for different values of μ are shown in Fig. 1. They all interest at one point C, of which we can convince ourselves by comparing the equalities $f_2(\mu, R_1, R_2) = f_3(\mu, R_1, R_2)$ and $f_2(\mu, R_1, R_2) = f_1(R_1, R_2)$, which lead to the same relation

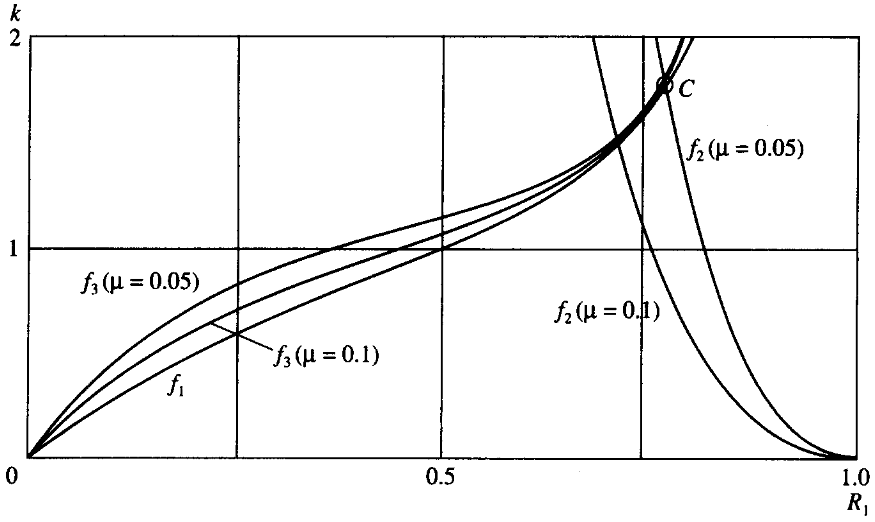

Fig. 1

$$(1-\mu)R_{2*}^2(1-R_{1*}^2) = \mu R_{1*}^2(1-R_{2*}^2) \tag{9}$$

defining the quantity R_{1*} at this point as a function of μ, and the corresponding value of k_* is found from one of the formulae

$$k_* = f_1(R_1, R_2), \quad k_* = f_i(\mu, R_{1*}, R_{2*}), \quad i = 2, 3$$

It follows from relation (9) that when $\mu \le 1/2$ we will have $R_{1*} \ge 1/2$ and, consequently, for the point C we always have $k \ge 1$. When $\mu \to 0$ this point is shifted upwards along the curve $k = f_1(R_1, R_2)$, approaching without limit to its asymptote $R_1 = 1$.

It follows from an analysis of inequalities (5), (6) and (7), (8) that stability of collinear libration points is possible either when inequalities (6) and (7) are simultaneously satisfied, i.e. when

$$k \ge f_1(R_1, R_2), \quad k < f_2(\mu, R_1, R_2), \quad k \le f_3(\mu, R_1, R_2) \tag{10}$$

or when inequalities (5) and (8) are simultaneously satisfied, i.e. when all the inequalities (10) are reversed.

The stability regions, defined by inequalities (10) and their opposites, for $\mu = 0.02$ are shown hatched in Fig. 2. The point C is the common boundary point of these regions. Note that when $k \le 1$ the inequalities, that are the opposite of inequalities (10), are incompatible. Each horizontal segment contained between the limits of the regions constructed, represents a whole set of stable collinear libration points, for the given value of k touching the curve $k = f_1(R_1, R_2)$ from the left (or the right), defining a family of triangular libration points [4] (Fig. 3). The coordinates of the left and right ends of this segment are respectively the roots of the equations

$$k = f_3(\mu, R_1, R_2), \quad k = f_1(R_1, R_2)$$

We will consider the problem of the stability at the point C itself, in which all the inequalities (10) become equalities. Representing the equilibrium condition in the form

$$\begin{aligned} [f_2(\mu, R_1, R_2) - k]Q_1 &= f_4(\mu, R_1, R_2) - k \\ f_4(\mu, R_1, R_2) &= (R_1 - \mu)R_2^2/\mu + 1 \end{aligned} \tag{11}$$

we conclude that a case is possible when it will be satisfied for any values of the reduction coefficient Q_1 (and, according to relation (2), the values of Q_2 corresponding to them), if the following equalities are simultaneously satisfied

$$k = f_2(\mu, R_1, R_2), \quad k = f_4(\mu, R_1, R_2) \tag{12}$$

which define the point of intersection of curves (12) in the k, R_1 plane.

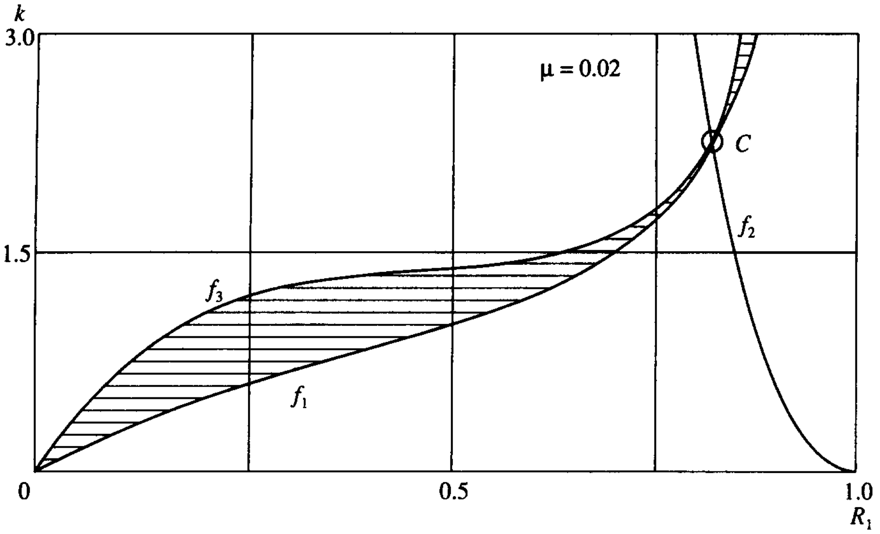

Fig. 2

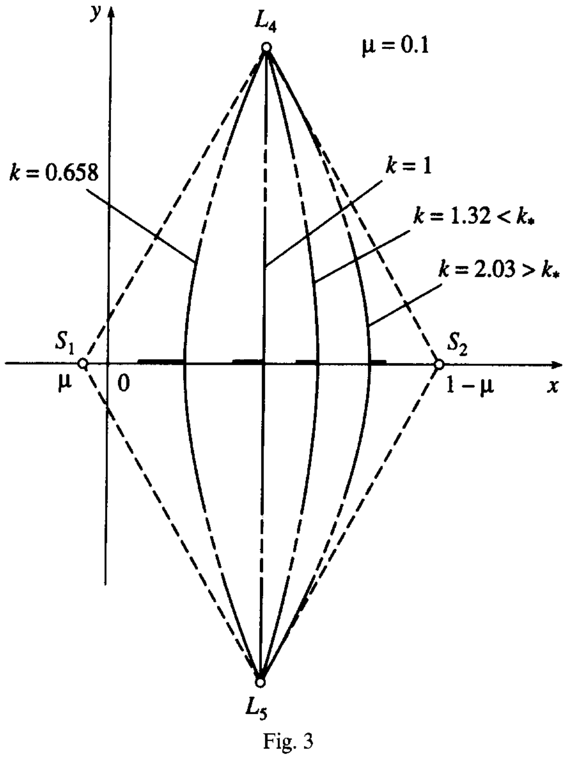

Fig. 3

It can be shown that the equation $f_2(\mu, R_1, R_2) = f_4(\mu, R_1, R_2)$ can be reduced to the form (9), i.e. the point C indicated above will again be the point of intersection of the curves (12). A unique collinear libration point belongs to each curve $k = f_1(R_1, R_2)$, for a fixed value of k defining the disposition of the family of triangular libration points for a fixed value of k (Fig. 3), and this unique collinear libration point is the point of intersection of this curve with the x axis. Since we have $Q_i = R_i^3$ ($i = 2, 3$) for the triangular libration point [3, 4], only particles with the same windage can be situated at these collinear libration points (it follows from the above investigation of the stability of triangular libration points [4] that for such particles stability will occur for any values of μ). However, for $k = k_*$ (i.e. at the point C of the stability region) particles with a different windage and a reduction factor connected only by relation (2) can lie at the same libration points, according to condition (1). However, only those whose reduction factors satisfy inequalities (3) will possess stability. Eliminating one of the reduction factors (for example,

Q_2) from the expression for A using relation (2) with $k = k_*$, and then k_* using, for example, the relation $k_* = f_1(R_1, R_2)$ and, finally, μ using relation (9), we will have

$$A = \frac{Q_1(1 - R_2^3) + R_1(R_2^3 - R_1^3)}{R_1 R_2(R_1^2 + R_2^2 - R_1^2 R_2^2)}$$

Inequalities (3) can now be written in the form

$$\frac{1}{9}R_1\left(8R_1^2 + \frac{R_1^3 - R_2^3}{1 - R_2^2}\right) \le Q_1 \le R_1^3 \tag{13}$$

Note that such a stable agglomeration of particles with different windages is only possible for values of $k_* \ge 1$ (this phenomenon has previously been pointed out for the case when $k_* = 1$ [5]), whence it follows, in turn, that such agglomeration can only occur when $R_1 \le 1/2$.

From the above analysis we can draw the following conclusions about the regions of stability of collinear libration points in configuration space (i.e. in the xy plane). When $k < k_*$ in addition to always stable collinear libration points into which triangular libration points transfer, for the same fixed value of k a family of stable collinear libration points also arises, situated in a segment of the x axis touching the curve $k = f_1(R_1, R_2)$ (Fig. 3) from the left (from the side of the body of greater mass). Here, at each point of this segment, there can be particles with only a unique pair of values of Q_1 and Q_2. The length of this segment increases as μ decreases, and it decreases as k increases, contracting to the point of intersection of the curve $k = f_1(R_1, R_2)$ with the x axis when $k = k_*$, where now particles with any values of Q_1 and Q_2 can occur at this libration point provided Q_1 satisfies inequalities (13).

When k is increased further, instead of the stable family of collinear libration points indicated, a new stable family occurs lying on the segment which now touches the same curve from the right (from the side of the body of smaller mass) (Fig. 3). The length of this segment for all values of μ and k is considerably less than that which touches the same curve from the left. It disappears altogether only when $k = \infty$, i.e. when $Q_1 = 1$, which occurs when there is no light flux on the body with mass $1 - \mu$.

We note one other interesting property of the region of stability of collinear libration points, situated on the segment touching the curve f_1 from the left for extremely small values of μ and large values of k. As a result of numerical calculations of the curves defining the stability regions (10), we established that when $\mu < \mu_{cr} = 0.0153\ldots$, two extremum points appear on the $k = f_3(\mu, R_1, R_2)$ curve (Fig. 4), as a result of which, for values of k obeying the inequality $k_{cr} < k < k_*$ (the quantity k_{cr} is uniquely defined by the quantity μ_{cr}), the segment of the x axis indicated above, which represents a family of stable collinear libration points, splits into two (in Fig. 5 these segments, like the family of stable triangular libration points, are denoted by thick lines). When k increases the instability gap separating these segments increases with a simultaneous reduction in the length of the separated segment. For a certain value of

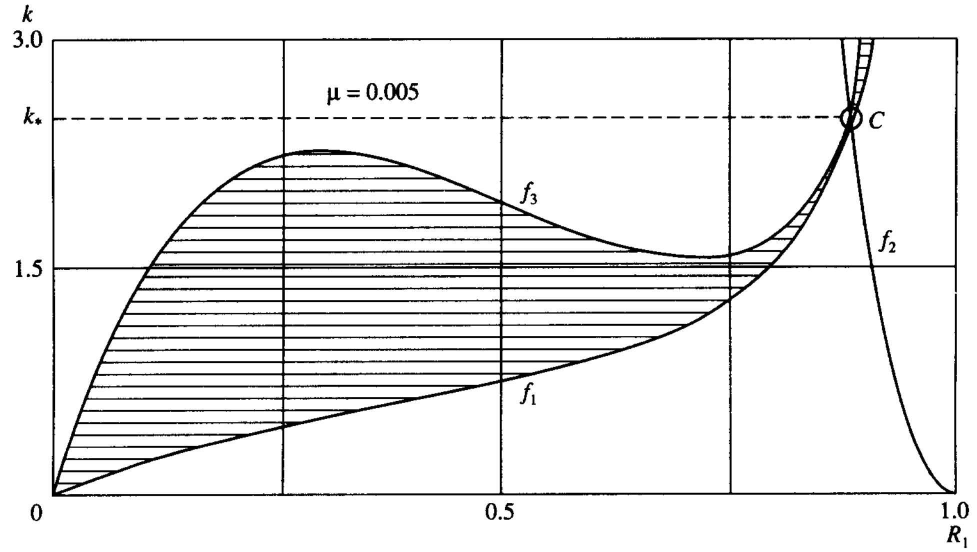

Fig. 4

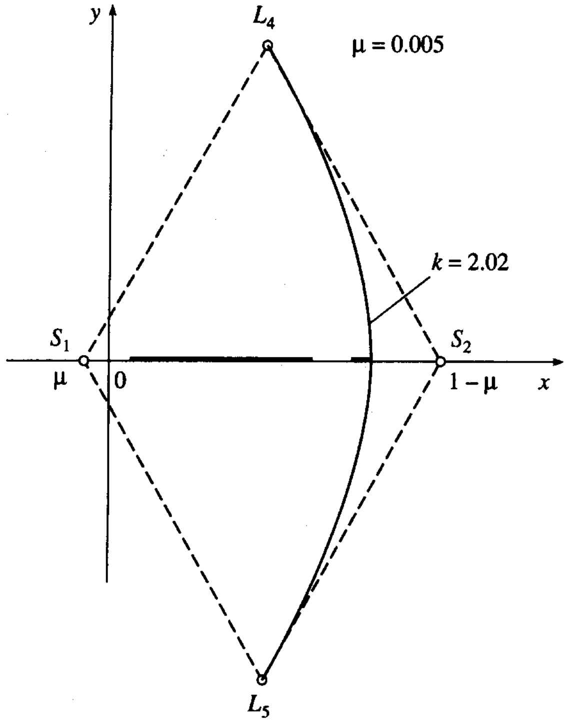

Fig. 5

$k = k_{max}$ it contracts to a point, after which (when $k > k_{max}$) it disappears completely, and the stability region in the xy plane takes the initial form. When there is a further reduction in μ it may turn out that $k_{max} > k_*$. This means that as k increases the family of libration points situated on the segment touching the curve $k = f_1(R_1, R_2)$ from the left initially disappears, and only after this (when there is a further increase in k) does the separated segment of the family of collinear libration points indicated above disappear.

This bifurcational value of k_{cr} is found from the condition $\partial f_3/\partial R_1 = 0$, which leads to a quadratic equation in μ. The quantity μ_{cr} is a multiple root of this equation. From the multiplicity condition we find the value $R_1^{cr} = 0.4753 \ldots$, corresponding to the value of μ_{cr}, which is the root of a high-degree algebraic equation. The corresponding value of k_{cr} is calculated from the formulae $k_{cr} = f_3(\mu, R_1, R_2)$, which gives $k_{cr} = 1.5074 \ldots$. Hence, the case considered only occurs when the body of smaller mass possesses a higher radiation power.

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